ECE 443/643 Homework 2

September 14, 2011

1. Let $\phi(t) = \text{sinc}(t/B)$ and $\phi_n(t) = \phi(t - nB)$ with $B > 0$ a constant real number and n an integer. Find $\langle \phi_n, \phi_m \rangle$ for all integers n and m, where $\langle x, y \rangle = \int_{-\infty}^{\infty} x(t)y^*(t)dt$ is the usual energy signal inner product.

2. Let $x(t) = e^{-t/\lambda}u(t)$ be the input to an LTI system with impulse response $h(t)$ and output $y(t)$.

   (a) Sketch $x(t)$.

   (b) Find the impulse response $h(t)$ so that $y(t_0) = E\{x\}$ and $y(t_0) \geq y(t)$ (In otherwords, $y(t)$ is maximized at t_0).

   (c) Given the $h(t)$ and $y(t_0) = y_0$ from the previous part, find a causal version $g(t) = h(t)u(t)$ of the impulse response so that the output of this new system is at least $0.9y_0$ at t_0. In otherwords, find the smallest t_0 so that $y_g(t_0) \geq 0.9y(t_0)$.

3. Evaluate the following:

   (a) The convolution $x(t) = (\Pi * \Pi)(t)$.

   (b) The Fourier transform $X(f) = \mathcal{F}\{x\}$.

   (c) The Fourier transform $\mathcal{F}\{e^{-|t|}\}$.

   (d) The Fourier transform $\mathcal{F}\{\cos(2\pi f_0 t)\}$.

4. An alternate definition of the Fourier transform is $X'(\omega) = \mathcal{F}'\{x\} = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$, which uses radian frequency instead of "Hertzian" frequency.

   (a) Find $X'(\omega)$ in terms of $X(f)$.

   (b) Find the extra conversion factor a in the synthesis equation $x(t) = a\int_{-\infty}^{\infty} X'(\omega)e^{j\omega t}d\omega$.

   (c) Find the extra conversion factor b in the energy integral required to keep the energy in x and X' equal: $\int_{-\infty}^{\infty} |x(t)|^2 dt = b\int_{-\infty}^{\infty} |X'(\omega)|^2 d\omega$.

5. (ECE644 only) Let

   $$x(t) = 0 \text{ if } t < a \text{ or } t > b$$

   (i.e. x is non-zero over an interval of length $b - a$) and

   $$y(t) = 0 \text{ if } t < c \text{ or } t > d$$

   (i.e. y is non-zero over an interval of length $d - c$). What is the maximum length of the interval for which the convolution $(x * y)(t)$ is non-zero?